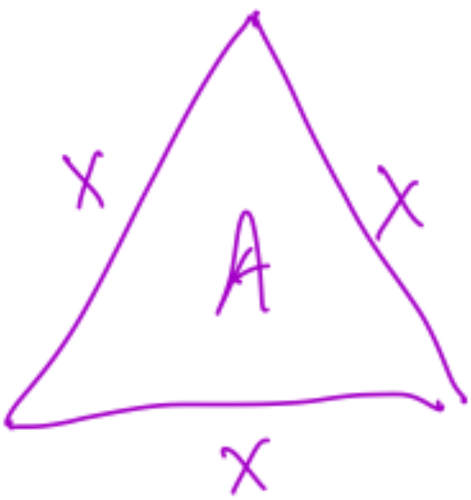
Question: An equilateral triangle is increasing in size at a rate of $10\,cm^2/min$. How fast are the sides increasing in length when the area is $50\,cm^2$?

$$10\frac{cm^2}{min} = \frac{dA}{dt}$$

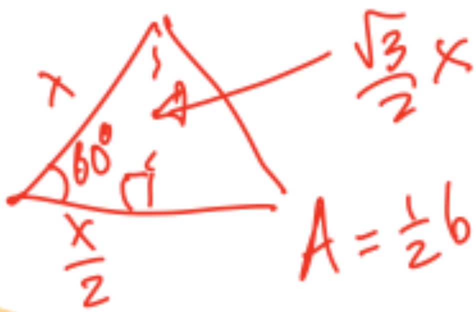
$$\frac{dx}{dt} = ?$$



Need
1. Equation with x & A in it.
2. Take deriv of both sides

$$A = \frac{\sqrt{3}}{4}x^2$$

Why?



$$A = \frac{1}{2}b\cdot h = \frac{1}{2}x\cdot\frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

$$A = \frac{\sqrt{3}}{4}x^2$$

$$\frac{dA}{dt} = \left(\frac{\sqrt{3}}{4}\right)\cdot 2x \cdot \frac{dx}{dt}$$

50

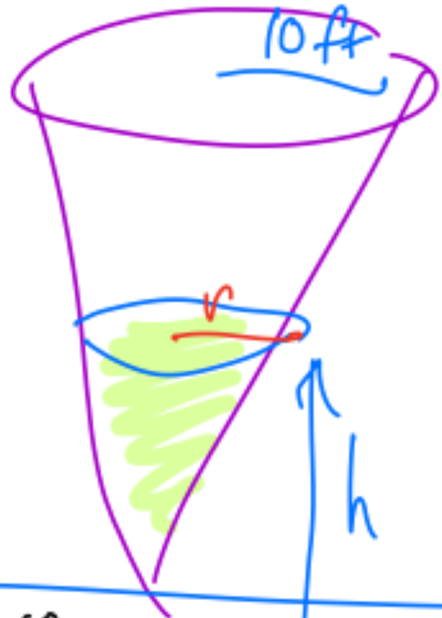
$A = 50\ cm^2$

$50 = \frac{\sqrt{3}}{4}x^2$

$\frac{200}{\sqrt{3}} = x^2$

$x = \sqrt{\frac{200}{\sqrt{3}}}$

$$\boxed{\frac{dx}{dt} = \_\_\_\_\ \frac{cm}{min}}$$



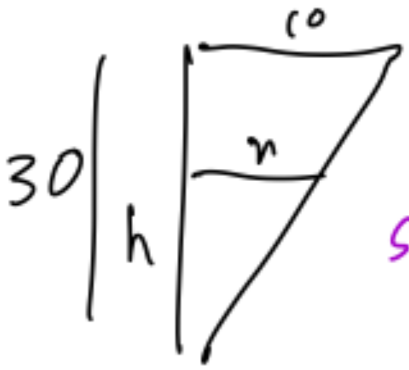
30 ft.

$\frac{dV}{dt} = -3\ ft^3/min$

$\boxed{\frac{dh}{dt} = ?}$

Need V in terms of h.

$h = 20\ ft.$



$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h$

$\boxed{V = \frac{\pi}{27}h^3}$

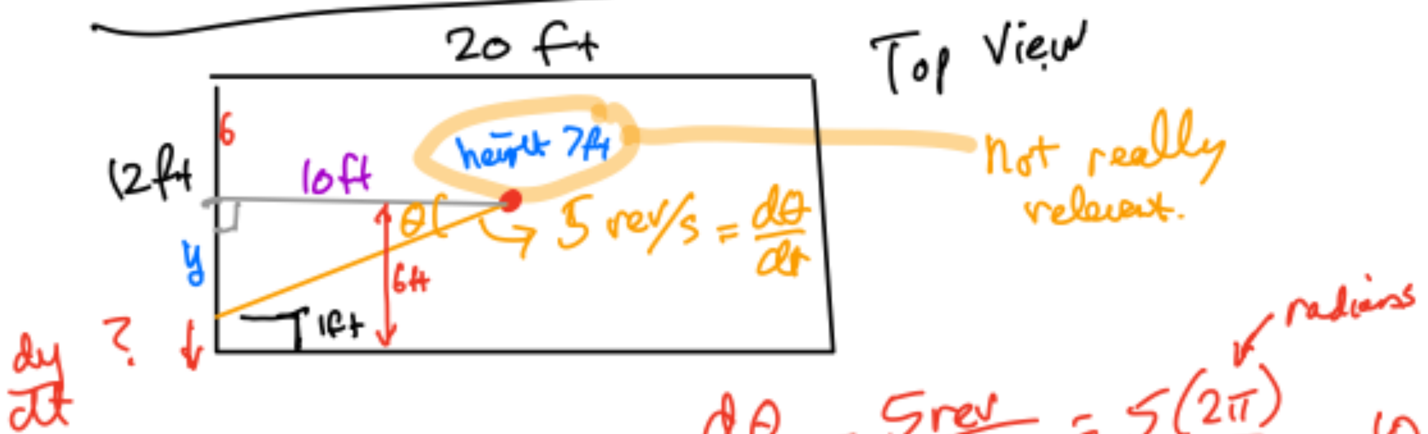
small big

$\frac{r}{h} = \frac{10}{30} = \frac{1}{3}$

$3r = h \Rightarrow r = \frac{1}{3}h$

FROM LAST TIME:

Example A laser flashlight is hanging from a ceiling and is rotating at a rate of 5 revolutions per second. It is at a height of 7ft, at the center of a room that is 12ft × 20ft. How fast is the flashlight's beam moving when it is at 1 ft from the corner of one of the 12ft walls?



radians

$$\frac{d\theta}{dt} = \frac{5\,\text{rev}}{s} = \frac{5(2\pi)}{s} = \frac{10\pi}{s}$$

Equation needed: should have θ & y in it.

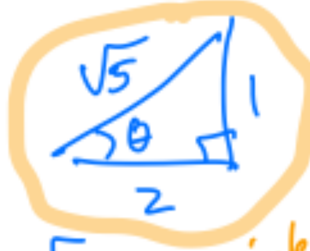
$$\tan\theta = \frac{y}{10}$$

derivaderize:
$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{10}\cdot\frac{dy}{dt}$$

$$(\quad)(10\pi) = \frac{1}{10}\frac{dy}{dt}$$

$y=5 \Rightarrow \tan\theta = \frac{5}{10} = \frac{1}{2}$

$\tan\theta = \frac{1}{2}$



$\theta = \arctan\left(\frac{1}{2}\right)$

$\sec\theta = \sec\left(\arctan\left(\frac{1}{2}\right)\right) = \frac{\sqrt{5}}{2}$

triangle trick.

$\sec^2\theta = \frac{5}{4}.$

$\frac{5}{4}\cdot 10\pi = \frac{1}{10}\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{5}{4}\cdot 10\pi \cdot 10 = \frac{500\pi}{4}$$

$$= 125\pi = \boxed{392.7\ \text{ft}/\text{s}}$$

(267.8 mph)

Linearization & the differential.

Differential: If $y=f(x)$

$$\frac{dy}{dx} = f'(x)$$

$$\Longrightarrow \boxed{dy = f'(x)\,dx}$$ differential

Think $\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x} = \text{slope} \approx f'(x)$

$$\boxed{\Delta y \approx f'(x)\,\Delta x}$$

Example: Find $\sqrt{4.002} - 2$ ← $\sqrt{4}$

If $y = \sqrt{x}$, this looks like

$$y_2 - y_1 \approx f'(x)\underbrace{(x_2 - x_1)}_{\Delta x}$$

$$f(4.002) - f(4) \approx f'(x)\cdot(4.002 - 4)$$

$$\underbrace{f(4.002) - f(4)}_{\Delta y} \approx f'(x)\underbrace{(.002)}_{\Delta x}$$

$$\underbrace{\sqrt{4.002} - 2}_{\Delta y} \approx f'(x)(.002)$$

$y = f(x) = \sqrt{x}$ at $x = 4$

$$f'(x) = \frac{1}{2}\cdot x^{-1/2} = \frac{1}{2}(4)^{-1/2} = \frac{1}{2\sqrt{4}} = \frac{1}{4}.$$

$$\Delta y = \sqrt{4.002} - 2 \approx \frac{1}{4}(.002)$$

$$= \frac{.002}{4} = \boxed{.0005}$$

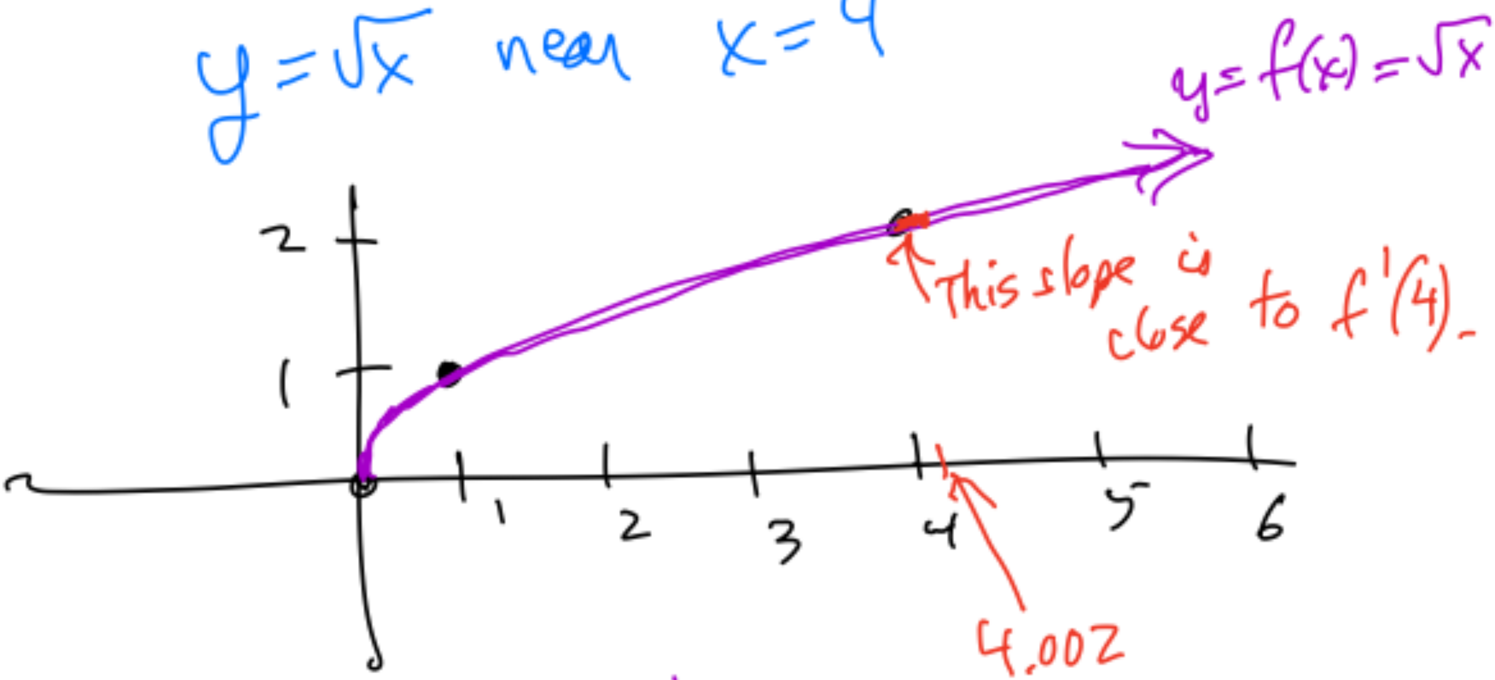
How close is it?

$$\sqrt{4.002} - 2 = 0.000499937 5\ldots$$

The differential gives a very good approximation to Δy!

What is going on here?

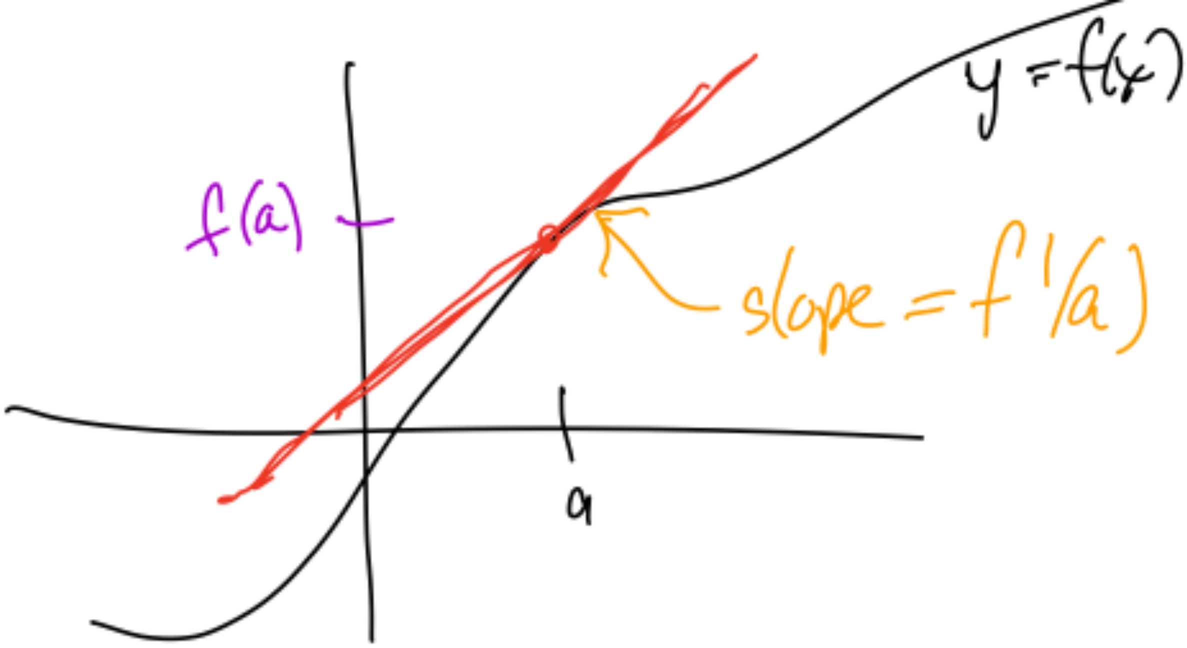
$y = \sqrt{x}$ near $x = 4$



Moral of the story:

$\boxed{\text{The tangent line to } y = f(x) \text{ at } x = a}$ is very close to $f(x)$ for x near a.

called The linearization of $f(x)$ at $x = a$.



Point-slope

$y - y_1 = m(x - x_1)$

$y - f(a) = (f'(a))(x - a)$

$$\boxed{y = f(a) + f'(a)(x-a)}$$

linearization of f(x) near x=a

$\approx f(x)$ near $x=a$.

$$\boxed{f(x) \approx f(a) + f'(a)(x-a)}$$

$\Delta y \approx f'(a)\,\Delta x \qquad (dy = f'(a)\,dx)$

Examples: Estimate $\frac{4}{1.007}$.

Let $f(x) = \frac{4}{x} = 4x^{-1}$

near $x=1$, $f(x) \approx f(1) + f'(1)(x-1)$

We want $f(x)$ where $x = 1.007$.

$$f'(x) = -\frac{4}{x^2} \overset{x=1}{=} -4$$

$$f(1.007) \approx 4 + (-4)(1.007 - 1)$$

$$= 4 - 4(.007)$$

$$= 4 - .028 = \boxed{3.972}$$

$$\boxed{\frac{4}{1.007} \approx 3.972}$$